EVERGREEN LIFSHITS-KHALATNIKOV QUASI-ISOTROPIC SOLUTION

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CONTENTS

Quasi-isotropic solution near cosmological singularity $t=0$

Lifshitz-Khalatnikov, 1960

$$ds^2 = dt^2 - \gamma_{lm}\,dx^l dx^m \qquad l,m = 1,2,3\ ; \quad p = \alpha\varepsilon$$

$$\gamma_{lm} = t^{2q} a_{lm} + t^2 b_{lm} + O(t^{4-2q}) \qquad q = \frac{2}{3(1+\alpha)},\quad 0 \le \alpha \le 1$$

$a_{lm} = a_{lm}(\vec{r})$

$b_{lm} = b_{lm}(\vec{r})$

$b \equiv a^{lm} b_{lm}$

$$8\pi G\varepsilon = \frac{3q^2}{t^2} - q\,b\,t^{-2q} + \dots$$

$$\frac{\delta\varepsilon}{\varepsilon} = -\frac{b}{3q}\,t^{2-2q} + \dots$$

Let P_l^m – the Ricci tensor for a_{lm}.

Then:

$$b_l^m = -\left(\frac{P_l^m}{1-q^2} + \frac{(q^2-2q+\frac{1}{3})\,P\,\delta_l^m}{2q(3-2q)(1-q^2)}\right)$$

$$b = -\frac{P}{2q(3-2q)}$$

$$u_l = \frac{q-2/3}{2q(1+q)}\,b_{,l}\cdot t^{3-2q} \rightarrow \text{potential flow}$$

a_{lm} – 3 arbitrary physical functions of $\vec{r}$ (1 – scalar perturbations, 2 – gravitational waves)

Magnetic part of the Weyl tensor $\neq 0$!

Two-fluid early-time quasi-isotropic solution

Contains a constant large-scale isocurvature mode

$p_l = k_l \varepsilon_l , \quad l = 1, 2 \quad -\frac{1}{3} < k_l \le 1$

$k_1 < k_2$

$$ds^2 = dt^2 - \gamma_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$\gamma_{\alpha\beta} = a_{\alpha\beta}\, t^{\frac{4}{3(k_2+1)}} + b\, a_{\alpha\beta}\, t^{\frac{2(3k_2-3k_1+2)}{3(k_2+1)}} + c_{\alpha\beta}\, t^2 + \dots$$

$$\varepsilon_2 = \frac{1}{6\pi G (k_2+1)^2 t^2} - \frac{b}{12\pi G (k_2+1)}\, t^{-\frac{2(k_1+1)}{k_2+1}} + \dots$$

$$\varepsilon_1 = \frac{(3k_2 - 2k_1 + 1)\, b}{12\pi G (k_2+1)^2}\, t^{-\frac{2(k_1+1)}{k_2+1}} + \dots$$

4 arbitrary functions

Valid until $\lambda \sim H^{-1} \sim t$

$\propto a^{-1}$

General quasi-de Sitter asymptote

(A. A. Starobinsky, Pisma v Zh ETF, 37 (1983) 55)

$$R_i^k - \frac{1}{2}\delta_i^k R = 8\pi G \epsilon_V \delta_i^k$$

$\epsilon_V = \mathrm{const} > 0$; $H^2 = 8\pi G \epsilon_V / 3$

$$ds^2 = dt^2 - \gamma_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$\gamma_{\alpha\beta} = e^{2Ht} a_{\alpha\beta}(\vec{r}) + b_{\alpha\beta}(\vec{r}) + e^{-Ht} c_{\alpha\beta}(\vec{r}) + O(e^{-2Ht})$$

$$t \to \infty$$

$$b_\alpha^\beta = \frac{1}{H^2}\left(P_\alpha^\beta - \frac{1}{4} P \delta_\alpha^\beta\right)$$

$$c_\alpha^\alpha = 0 \ ; \quad c_{\alpha\ ;\beta}^{\beta} = 0.$$

P_α^β – the 3-space curvature tensor (Ricci) constructed from $a_{\alpha\beta}(\vec{r})$.

4 physical arbitrary functions of $\vec{r}$:

$a_{\alpha\beta}$: $6 - \underbrace{3 - 1}_{\text{gauge freedom}} = 2$

$c_{\alpha\beta}$: $6 - 4 = 2$

$\to 4$

After the phase transition: $t = t_0(\vec{r})$

$$\gamma_{\alpha\beta} = e^{2Ht} a_{\alpha\beta}(\vec{r}) + \dots = e^{2H(t-t_0)} e^{2Ht_0} a_{\alpha\beta}(\vec{r}) + \dots \to e^{2Ht_0} a_{\alpha\beta}(\vec{r})\, t + \dots$$

$$\to a(t - t_0(\vec{r})) \cdot e^{2Ht_0} a_{\alpha\beta}(\vec{r}) + \dots \to$$

Quasi-isotropic solution and δN-formalism

Inflation produces initial conditions for the quasi-isotropic solution

Let $$N = N(x) = \int_{t_i}^{t_f} H(t,x)\,dt$$

is the local duration of inflation ($|\dot{H}| \ll H^2$) in terms of the number of e-folds ($H \equiv \dot{a}/a$).

Then $$a_{\alpha\beta} = e^{2\zeta(x)}\left(\delta_{\alpha\beta} + \underbrace{h_{\alpha\beta}}_{\substack{\text{small TT part} \\ (\text{GW})}}\right)$$

$$\delta\zeta(x) = \delta N = \frac{\delta N}{\delta\varphi}\,\delta\varphi(x)$$

Observations:

$$\underbrace{\frac{\Delta T(\theta,\varphi)}{T}}_{\substack{\text{large-angle} \\ \ell < 50}} \approx -\frac{1}{5}\delta\zeta(x)\Big|_{LSS}$$

Critical two-fluid case

$$k_1 = -\frac{1}{3} \rightarrow \text{"stringy gas"}$$

$$k_2 = \frac{1}{3} \rightarrow \text{for simplicity only}$$

$$\gamma_{\alpha\beta} = a_{\alpha\beta}(x)\left(t + b(x)t^2\right) + c_{\alpha\beta}(t,x)$$

$$c_{\alpha\beta} \propto t^2 \ln bt \quad (\text{though } c \equiv c_\alpha^\alpha \propto t^2)$$

Breaking of large-scale factorization at late times!

Characteristic time variable for $t \to \infty$:

$$\eta = \int \frac{dt}{a(t)} \propto \ln t$$

Quasi-isotropic stage may be very long since $\lambda \propto t$